Από Leybourne, The Mathematical Questions proposed in Ladies Diary 1704-1816

Έκδοσης 1817 που επαναλαμβάνει το υλικό στο Ladies Diary του 1745-46.

THE

MATHEMATICAL QUESTIONS,

PROPOSED IN THE

LADIES' DIARY,

AND THEIR ORIGINAL ANSWERS,

TOGETHER WITH SOME NEW SOLUTIONS,

FROM ITS COMMENCEMENT IN THE YEAR

1704 TO 1816.

IN FOUR VOLUMES.

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1817.

Σελίς 373:

V. QUESTION 260, *by Mr.* N. Farrer.

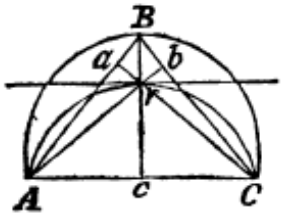
In an oblique-angled triangular grove, one of whose sides is 20 chains, and the angle opposite thereto 78° 45′, if a perpendicular be let fall from each angle to its opposite side, they intersect at a

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fountain within the grove, whose nearest distance to the given side is 8·19 chains: Quere, its distance from each of the other sides, by a simple equation?

Answered by Mr. Heath.

It is evident by the data that the triangle must be isosceles. In the △ABC, the ∠A = ∠C, or the ∠B must be the given angle; and either way the ∠*a*rB = ∠*c*rB, or ∠A*r*C is given = 101° 15′ opposite to the side given = 20 chains, on which describing a segment of a circle to contain it, and drawing the parallel distance 8·19 (or rather 8·2 as it should be) chains = *cr*, and it will be found a maximum (as is also proved by trigonometry), therefore the data are *rc*, AC = cc, and ∠B, ∠A = ∠C, and ∠BA*b*; whence follow by trigonometry A*r* = C*r* = 12·925, *ar* = *rb* = 2·5215 (AB = BC = 15.64) chains required.



Additional Solution.

The meaning of this prob. is thus: In a triangle we have given the base (AC), the vertical angle (ABC), and the distance (*cr*) of the base from the common point of intersection of three lines drawn from the three angles perpendicular to the opposite sides; to determine the triangle.

We are not at liberty to suppose the triangle isosceles, for that would be introducing a condition too much into the problem; and whether it be isosceles or of any other form, can only appear from the construction or calculation.

Construction. On the given base describe two segments of circles, the one ABC to contain the given vertical angle, and the other A*r*C to contain its supplement; parallel to AC, and at the given distance of the point from it, draw a line to cut or touch A*r*C in *r*; then through *r* draw *crb* ⊥ AC, and B will be the vertex of ABC the triangle required.

For, through *r* drawing A*r*b and c*r*c, since by the construction AC is the given base, ABC the given vertical angle, and *rc* the given distance, we have only to prove that the angles at *a* and *b* are right angles. Now by the construction it appears that A*r*C, ABC are segments of equal circles, and that the two segments together make up a whole circle; then the ∠ABr or *a*B*r* = ∠A*c*r as standing on equal segments A*r*, and the opposite angles *r*uB, *cr*c are also equal; ∴ the third angles are equal, that is ∠*a* = ∠*c* = a right angle. In the like manner *b* is proved to be a right angle.

Scholium. Another method of construction might be by first finding the point *r* as before, through which draw the indefinite lines

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ArB, *cra*, and perpendicular to them the lines *cb*B, A*a*B.——And then we should have to prove that these last two lines and *cr*, produced, meet in the same point B, and that ABC is = the given angle.

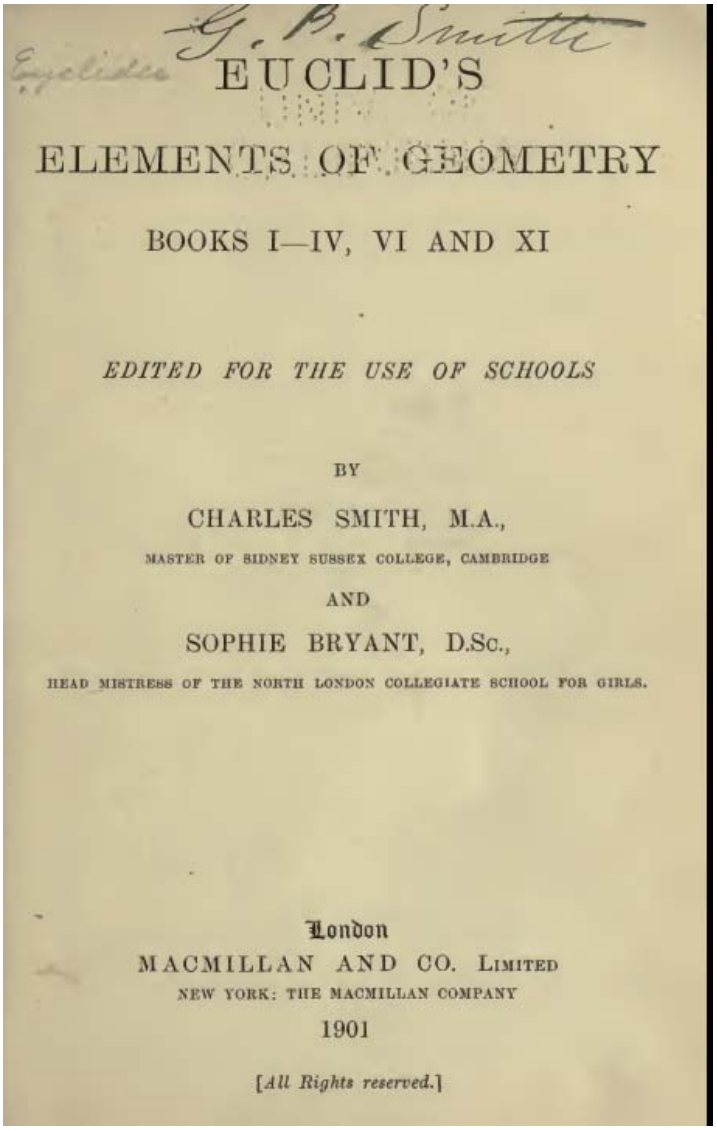
Corollary 1. The equal opposite angles A*ra*, *crb*, being the supplements of the ∠A*r*C, are each = the ∠ABC, which is also the supplement of A*r*C by the construction.

Corollary 2. Hence also the ∠ BA*r* = BC*r*, and the four triangles BA*b*, *a*A*r*, BC*a*, *bcr* are all similar.

The Method of Calculation will be, first to calculate the angles *r*AC, *r*CA, of the △A*r*C, by prob. V. Simpson's Algebra, and then the segments AC, CC; then to each of these two angles adding the ∠BA*b* or BC*a* = the comp. of ∠ABC, there will be had the ∠BAC and BCA; from which, and the segments AC, CC, the two hypothenuses AB, BC are easily got, and will come out 15·386 and 16·127, the angles at A and C being 52° 16′ and 48° 59′. H.

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Smith, Bryant, Euclides Elements of Geometry, 1901:



G. B. Smith

Euclides

EUCLID'S

ELEMENTS OF GEOMETRY

BOOKS I—IV, VI AND XI

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Σελίς 228:

It should be noticed that, if O be the orthocentre of the $\triangle$ ABC, then A, B, C will be the orthocentres of the $\triangle^s$ BOC, COA, AOB respectively, the feet of the perpendiculars for all four triangles being the points D, E, F.

The $\triangle$ DEF, whose angular points are the feet of the perpendiculars of the $\triangle$ ABC is called the **pedal triangle** of the $\triangle$ ABC.

It should also be noticed that, since the angles BOC and BAC are supplementary, the $\odot^s$ BOC and BAC are equal; and similarly each of the $\odot^s$ COA, AOB is equal to the $\odot$ ABC.

Thus, *if O is the orthocentre of the triangle ABC, the four circles BOC, COA, AOB, ABC are all equal.*

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Conversely, *if ABC be any triangle and O a point such that the circles BOC, COA, AOB are equal, then each is equal to the circle ABC and O is the orthocentre of the triangle ABC.*